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To:

From:

Subject: Variation of Photographic Density with Numerical Aperture STATINTL
for a Nonscattering Emulsion

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Introduction

It is well known that photographic density depends on the numerical apertures of both the source and the detector, with high numerical apertures, in general, providing lower densities. This decrease is attributed to scattering in the emulsion. Very fine grain emulsions will scatter little light thereby possibly allowing a second effect to be observed. This effect is an increase in density with increasing numerical aperture, due simply to the longer path lengths taken by the light through the three-dimensional emulsion. It is the purpose of this memorandum to determine the magnitude of this density increase for a theoretical emulsion consisting of a homogeneous partially absorbing medium of constant thickness and specific index of refraction.

Calculation of Density

Consider a three-dimensional emulsion of thickness (t) containing a homogeneous light absorbing medium, illuminated by a diffuse source whose intensity (watts/ster.) is isotropic. Only the light lying within an angle θ_0 with the normal to the emulsion is collected by a detector (Figure 1). The transmission (T) of the emulsion is defined as
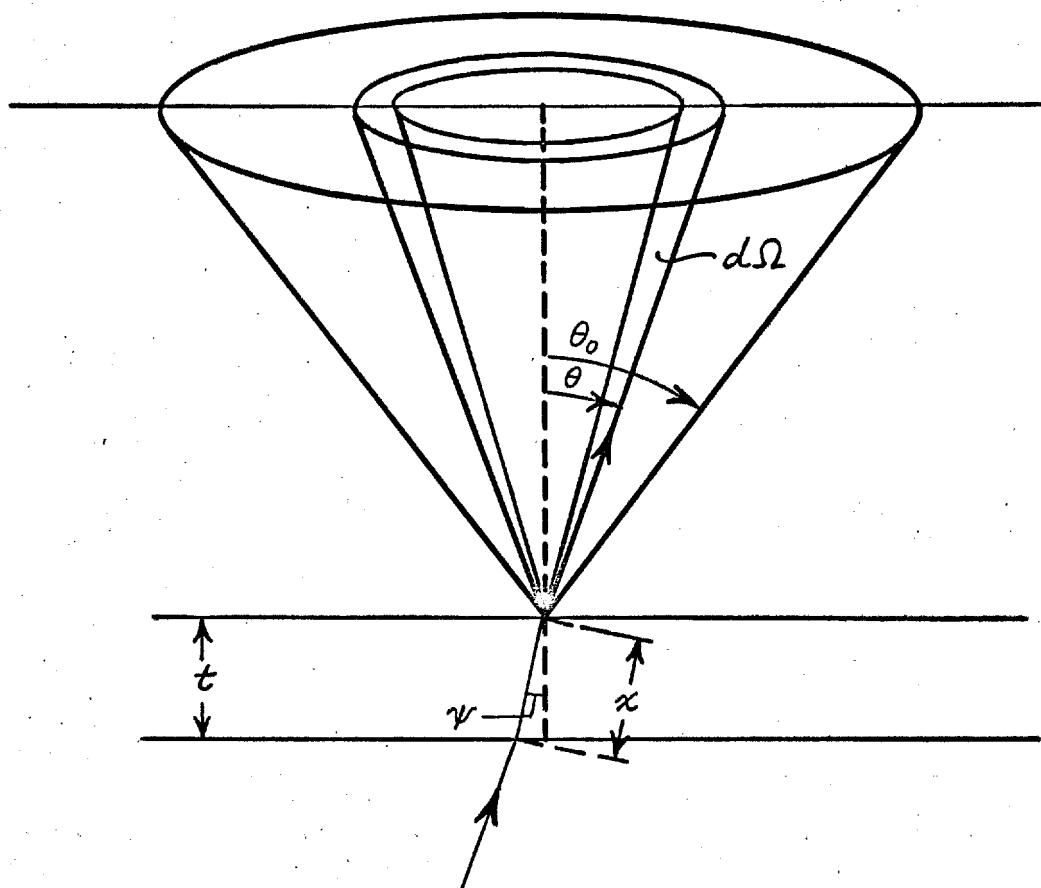
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Figure I   GEOMETRY OF RAY PROPAGATION

the ratio of the flux $(\bar{\phi})$ transmitted by the emulsion (within the angle θ_0) to the flux $(\bar{\phi}_0)$ with the emulsion removed:

$$T = \frac{\bar{\phi}}{\bar{\phi}_0}$$. (1)

The flux $\bar{\phi}_0$ can be written:

$$\bar{\phi}_0 = \int_{\theta=0}^{\theta_0} \phi_0(\theta) \, d\Omega(\theta)$$ (2)

where: Ω = solid angle (Figure 1)

              ϕ_0 = source intensity (watts/ster.)

For isotropic ϕ_0 :

$$\bar{\phi}_0 = \phi_0 \int_{\theta=0}^{\theta_0} d\Omega(\theta)$$ (3)

The differential solid angle $d\Omega$ is given by:

$$d\Omega = 2\pi \sin\theta \, d\theta$$ (4)

Combining Equations (3) and (4) and integrating:

$$\bar{\phi}_0 = 2\pi \phi_0 (1 - \cos\theta_0)$$ (5)

In like manner, the flux $\bar{\phi}$ is written:

$$\bar{\phi} = \int_{\theta=0}^{\theta_0} \phi(\theta) \, d\Omega(\theta) = 2\pi \int_0^{\theta_0} \phi(\theta) \sin\theta \, d\theta$$ (6)

where: $\phi(\theta)$ = transmitted light intensity (watts/ster.)

For the absorbing medium under consideration $\phi(\theta)$ is:

$$\phi(\theta) = \phi_0 e^{-\alpha(t)x(\theta)} \tag{7}$$

where: $x(\theta)$ = path length in medium

$\alpha(t)$ = absorption coefficient

t = emulsion thickness

It is worth mentioning that in writing α as a function of t, the distribution of the absorbing medium in the thickness direction is not assumed to be constant. The path length (x) in the absorbing medium is given by:

$$x(\theta) = t \sec \psi = \frac{nt}{\sqrt{n^2 - \sin^2\theta}} \tag{8}$$

where: n = index of refraction of emulsion

ψ = refraction angle

Eliminating $x(\theta)$ and $\phi(\theta)$ between Equations (6), (7), and (8) yields:

$$\phi = 2\pi\phi_0 \int_0^{\theta_0} e^{-\frac{n\beta}{\sqrt{n^2 - \sin^2\theta}}} \sin\theta \, d\theta \tag{9}$$

where: $\beta \equiv t\alpha(t)$

In the hope of finding a more tractable form for the integrand, the following transformation is used:

$$\xi = (n^2 - \sin^2\theta)^{-1/2} \tag{10}$$

Thus:

$$\bar{\phi} = 2\pi\,\phi_0 \int_{1/n}^{(n^2 \sin^2\theta_0)^{-1/2}} \frac{e^{-n\beta\xi}}{\xi^2\sqrt{1-(n^2-1)\xi^2}}\, d\xi \tag{11}$$

Unfortunately, a closed form solution to the integral of Equation (11) is not known to this writer. However, in order to obtain some idea of the magnitude of the decrease of $\bar{\phi}$ with θ_0 , n is set equal to unity:

$$\bar{\phi} = 2\pi\,\phi_0 \int_1^{\sec\theta_0} \xi^{-2} e^{-\beta\xi}\, d\xi \tag{12}$$

The integral of Equation (12) is known:

$$\int_1^{\sec\theta_0} \xi^{-2} e^{-\beta\xi}\, d\xi = e^{-\beta} + \beta Ei(-\beta) - e^{-\beta\sec\theta_0}\cos\theta_0 - \beta Ei(-\beta\sec\theta_0) \tag{13}$$

Eliminating the fluxes $\bar{\phi}$ and $\bar{\phi}_0$, and the integral between Equations (1), (5), (12), and (13), yields the transmission:

$$T = \frac{e^{-\beta} - e^{-\beta\sec\theta_0}\cos\theta_0 + \beta\left[Ei(-\beta) - Ei(-\beta\sec\theta_0)\right]}{1-\cos\theta_0} \tag{14}$$

The specular transmission (T_0) is found from Equation (14) when $\theta_0 = 0$:

$$T_0 = e^{-\beta} \tag{15}$$

In terms of T_0 , Equation (14) is written:

$$T = \frac{T_0 - T_0^{\sec\theta_0}\cos\theta_0 - (\ln T_0)\left[Ei(\ln T_0) - Ei(\ln T_0^{\sec\theta_0})\right]}{1-\cos\theta_0} \tag{16}$$

This relation is shown in Figure 2 for three density levels, where density (D) and numerical aperture ($N.A.$) are plotted rather than T and θ_o :

$$
\left.
\begin{aligned}
D &\equiv -\log_{10} T \\
N.A. &\equiv \sin \theta_o
\end{aligned}
\right\}
\tag{17}
$$

Since the index of refraction was set equal to unity, the curves represent the largest deviations that could be expected from a nonscattering emulsion. This can be understood when it is realized that refractive indices greater than one result in smaller angles of refraction (γ) and hence shorter path lengths (x) through the emulsion.
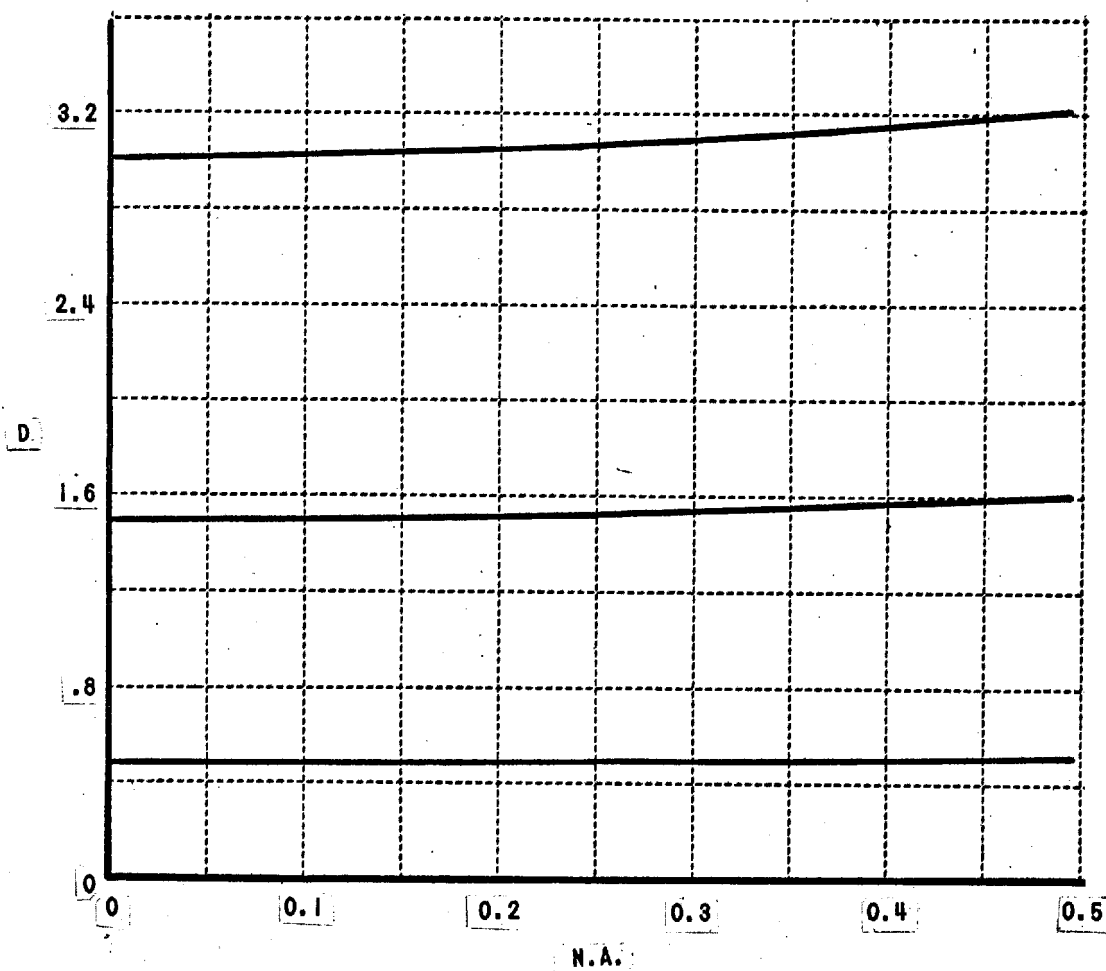
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Figure 2   VARIATION OF DENSITY WITH NUMERICAL APERTURE